

# HOW TO... Generate chaos in Excel

Excel is widely known for bringing order and structure to a variety of tasks, but here Mike Bedford explains how it can be used to get to grips with chaos theory

It's been said that the tiniest variations in atmospheric pressure caused by a butterfly flapping its wings in Japan could eventually cause a hurricane in the Caribbean. The butterfly effect – the notion that tiny variations in a system can generate large-scale variations – is often attributed to chaos theory.

Chaos theory is an intriguing field of mathematics with close links to fractals such as the Mandelbrot Set. However, although the Earth's atmosphere is a good example of a chaotic system, the equations that describe it are complicated. Still, there are other ways to visualise chaos theory. Here we'll show you how to bring it to life on your computer screen using only a copy of Excel and, in so doing, generate the fascinating Bifurcation Diagram.

For this walkthrough, we're going to use a different chaotic system – a simple model of the population growth of fruit flies. It is summed up in the following equation:

$$x_n = x_{n-1} + ax_{n-1}(1-x_{n-1})$$

The equation shows how the population ( $x_n$ ) at the current time is related to the population some time ago ( $x_{n-1}$ ) and to a growth rate ( $a$ ). The  $1-x_{n-1}$  part of the equation reflects the fact that there is a maximum population the environment can support, perhaps because of limited food supply. In mathematical language the population is 'normalised to 1', which means

1 is the maximum supported population, but you can multiply the values up, perhaps by a few million, to get the real population.

**1** Download the Chaos Theory.xls file from the cover CD and open it in Excel. Enter 1 in cell A6 and 0.1 in B6. These are the initial values for the time,  $n$ , and the population,  $x$ , respectively. Type the formula  $=A6+1$  into cell A7 and  $=B6+\$C\$3*B6*(1-B6)$  into B7 to augment the time and work out what the population will be at the next time interval. The new population that appears in B7 is 0.118. To see how the population changes over time, click on cell A7 to select it, drag across to cell B7 and then down to B55. This will highlight cells A7 to B55. Then select Fill, Down from the Edit menu.

If you now scan down column B, you'll see that the population gradually rises closer to 1. Extending the formulae further down the columns would show that once the population reaches 1 it goes no higher.

**2** It's far easier to see trends graphically, so we'll create a line graph showing population growth. Select A6:B55 and click on the Chart Wizard icon. Select XY Scatter as the Chart type on the left and then on the bottom right-hand option under Sub-chart type, on the right. Click Next and then Next again. Enter Time into the Value (X) axis box and Population into the Value (Y) axis box before clicking on Finish.

**3** On the graph that appears, click on the Series 1 box and press the Delete key on your keyboard to remove it. The graph shows values of time up to 60, but we are really only interested in values up to 50. Right-click on the horizontal axis and select Format Axis from the drop-down menu. Select the Scale tab and type 50 into the Maximum box. Then click OK. Right-click on any part of the graph and choose the Format option to make changes such as colour. Use the resize handles (the little black squares around the outside of the graph) to make the graph a bit bigger.

**4** You can now try out different values of the growth constant  $a$  to see how it affects the population growth. First, try a lower value. Type 0.1 into C3. The trend is similar except that the growth is slower. Now type 1.0 into C3. The population still flattens out at 1 but it reaches this value much more quickly.

Next enter 2.2 into C3. This time, something quite different happens. The population no longer settles down to a final value, but instead oscillates between two states. This is a 'boom and bust' phenomenon. The population grows so rapidly that it gets larger than the environment can support, after which there is a drop due to competition for limited resources. Then it starts all over again. Type 2.5 into C3, and you'll find that it oscillates between four values. With 2.57 it oscillates between eight states.



